

Vedic Method of Finding Square Roots

The procedure for extracting the square root of a number is similar to the method of straight division except that, in this case, the Divisor should be exactly double the first digit of the square root. Please note that, as a single digit can never be more than 9, no Divisor greater than 18 is ever necessary.

Example 1) What is $\sqrt{622521}$?

We will solve this question in two different ways: first, without using vinculum numbers; second, with vinculum numbers.

Practical Method 1 (without using vinculum numbers)

Step1:

We form distinct 'two-digit' groups from the right, and separate the last (far left) group with a forward slash. Then we leave a single blank row space for writing down the remainders just as in the straight division method. After this, we draw a vertical line at the left side of the number. Here we will write the Divisor.

	62 / 25 21

Step 2:

In our example, the first group, located on the far left, separated by a forward slash, is 62. We ask ourselves, what is the maximum square that can be subtracted from this group? The answer is 49. Now we write the square root of 49 or 7 below the horizontal line and this is the very first digit in the square root of the answer. The forward slash is written directly underneath. The Divisor which is exactly double the first digit of the square root, or in this case 2×7 or 14, is written to the left of the vertical line.

	62 / 25 21
14	_____
	7 /

Step 3:

We write the remainder $62 - 49$ or 13 above the horizontal line ahead of the 2, as in the straight

division method.

	6 2 / 2 5 2 1
14	13
	7 /

Step 4:

The gross dividend (G.D.), after prefixing the remainder 13 to the 2, is now 132. Without subtracting anything from it, we simply divide the 132 by 14 giving: 9 remainder 6 which is also equivalent to 8 remainder 20. *(Please refer to the Quotient/Remainder section in Fundamentals to refresh your memory)*

At this moment in time, we will use the latter quotient/remainder pair just to avoid unnecessary complications.

	6 2 / 2 5 2 1
14	13 20
	7 / 8

Step 5:

The next gross dividend (G.D.) is 205. Before dividing by 14 we find the net dividend (N.D.) by subtracting the duplex of the second square root digit *(in this example it is 8)* from the G.D.

i.e. N.D. = 205 - Duplex(8)

$$\text{N.D.} = 205 - 8^2 = 205 - 64 = 141$$

Now, $141 \div 14 = 10 \text{ rem } 1$ or $9 \text{ rem } 15$. Again, to avoid complications we will use the latter quotient/remainder pair i.e. Q = 9 and R = 15. We then write the quotient and remainder in their respective places.

	6 2 / 2 5 2 1
14	13 20 15
	7 / 8 9.

Admittedly, the original number was actually a perfect square and as there are three 'two-digit' groups, the square root has exactly three digits, and so the calculation is complete, giving **789 as the square root of 622521.**

If, however, the original number that was given was not a perfect square we would put a decimal

point after the third digit and proceed in a similar fashion. So, just to verify that 789 is the square root of 622521 we will continue with the rest of the computation.

Step 6:

$$\text{G.D.} = 152 \text{ so N.D.} = 152 - \text{Duplex}(8\ 9) = 152 - (2 \times 8 \times 9) = 152 - 144 = 8$$

$$8 \div 14 = 0 \text{ rem } 8 \text{ i.e. quotient} = 0 \text{ and remainder} = 8$$

	6 2 / 2 5 2 1
	13 20 15 8
14	7 / 8 9. 0

Step 7:

$$\text{The next and last G.D.} = 36 \text{ so N.D.} = 36 - \text{Duplex}(7\ 6\ 0) = 36 - (2 \times 7 \times 0 + 6^2) = 36 - 36 = 0$$

$$0 \div 0 = 0 \text{ rem } 0 \text{ i.e. Q} = 0 \text{ and R} = 0$$

	6 2 / 2 5 2 1
	13 20 15 8 0
14	7 / 8 9. 0 0

The work is completed, 622521 is a perfect square!

As mentioned before, if the given number is not a perfect square we will get the answer in decimal form, and we will give some practical examples of how to do this later on.

Now, we will do the same calculation but sometimes using vinculum numbers wherever necessary.

Practical Method 2 (using vinculum numbers)

Q1.) What is $\sqrt{6\ 2\ 2\ 5\ 2\ 1}$?

Step1: (As before)

As usual we form distinct 'two-digit' groups from the right, and separate the last (far left) group with a forward slash. Then we leave a single blank row space for writing down the remainders just as in the straight division method. After this, we draw a vertical line at the left side of the number. Here we will write the Divisor.

	6 2 / 2 5 2 1

Step 2: (As before)

In our example, the first group, located on the far left, separated by a forward slash, is 62. We ask ourselves, what is the maximum square that can be subtracted from this group? The answer is 49. Now we write the square root of 49 or 7 below the horizontal line and this is the very first digit in the square root of the answer. The forward slash is written directly underneath. The Divisor which is exactly double the first digit of the square root, or in this case 2×7 or 14, is written to the left of the vertical line.

	6 2 / 2 5 2 1
14	_____
	7 /

Step 3: (As before)

We write the remainder $62 - 49$ or 13 above the horizontal line ahead of the 2, as in the straight division method.

	6 2 / 2 5 2 1
14	13 _____
	7 /

Step 4: (As before)

The gross dividend (G.D.), after prefixing the remainder 13 to the 2, is now 132. Without subtracting anything from it, we simply divide the 132 by 14 giving: 9 remainder 6 which is also equivalent to 8 remainder 20. *(Please refer to the Quotient/Remainder sub-section in 'Fundamentals' to refresh your memory)*

At this moment in time though, we will use the former quotient/remainder pair i.e. $Q = 9$ and $R = 6$

	6 2 / 2 5 2 1
14	13 6
	7 / 9

Step 5:

The next gross dividend (G.D.) is 65. Before dividing by 14 we find the net dividend (N.D.) by subtracting the duplex of the second square root digit (*in this example it is 9*) from the G.D.

i.e. $N.D. = 65 - \text{Duplex}(9)$

$$N.D. = 65 - 9^2 = 65 - 81 = \overline{16} \text{ or } -16$$

We have a negative net dividend! But this is no problem....let's proceed.

Now, $-16 \div 14 = \overline{1} \text{ rem } \overline{2}$. We then write the quotient and remainder in their respective places.

	6 2 / 2 5 2 1
14	13 6 $\overline{2}$
	7 / 9 $\overline{1}$.

Step 6:

$$G.D. = \overline{22} \text{ so } N.D. = \overline{22} - \text{Duplex}(9 \overline{1}) = \overline{22} - (2 \times 9 \times \overline{1}) = \overline{22} - (\overline{18}) = (\overline{18}) - (\overline{18}) = 0$$

$$0 \div 0 = 0 \text{ rem } 0 \text{ i.e. } Q = 0 \text{ and } R = 0$$

	6 2 / 2 5 2 1
14	13 6 $\overline{2}$ 0
	7 / 9 $\overline{1}$. 0

The next and last G.D. = 01 so $N.D = 01 - \text{Duplex}(9 \overline{1} 0) = 01 - (2 \times 9 \times 0 + \overline{1}^2) = 01 - 01 = 0$

$$0 \div 0 = 0 \text{ rem } 0 \text{ i.e. } Q = 0 \text{ and } R = 0$$

and so....

	6 2 / 2 5 2 1
14	13 6 2 0 0
	7 / 9 1. 0 0

Now, we have the answer $79\bar{1}$ which is the same as 789. So we have found the $\sqrt{622521}$ with and without using the vinculum.

Example 2) What is $\sqrt{331776}$?

Practical Method 1

Step1:

As before, we form distinct 'two-digit' groups from the right, and separate the last (far left) group with a forward slash, leaving a single blank row space for writing down the remainders, just as in the straight division method. After this, we draw a vertical line at the left side of the number. Here we will write the Divisor.

	3 3 / 1 7 7 6

Step 2:

In our example, the first group, located on the far left, separated by a forward slash, is 33. Again, we ask ourselves, what is the maximum square that can be subtracted from this group? The answer is 25. Now we write the square root of 25 or 5 below the horizontal line and this is the very first digit in the square root of the answer. The forward slash is written directly underneath. The Divisor which is exactly double the first digit of the square root, or in this case 2×5 or 10, is written to the left of the vertical line.

	3 3 / 1 7 7 6
10	5 /

Step 3:

We write the remainder $33 - 25$ or **8** above the horizontal line ahead of the 1, as in the straight division method.

		33	$/$	17		76
				8		
10						
		5	$/$			

Step 4:

The gross dividend (G.D.), after prefixing the remainder 8 to the 1, is now 81. Without subtracting anything from it, we simply divide the 81 by 10 giving: 8 remainder 1 which is also equivalent to 7 remainder 11. *At this moment time, we will use the latter quotient/remainder pair just to temporarily avoid unnecessary complications.*

		33	$/$	17		76
				811		
10						
		5	$/$	7		

Step 5:

The next gross dividend (G.D.) is 117. Before dividing by 10 we find the net dividend (N.D.) by subtracting the duplex of the second square root digit (*in this example it is 7*) from the G.D.

i.e. $N.D. = 117 - \text{Duplex}(7)$

$$N.D. = 117 - 7^2 = 117 - 49 = 68$$

Now, $68 \div 10 = 6 \text{ rem } 8$ i.e. $Q = 6$ and $R = 8$. We then write the quotient and remainder in their respective places.

		33	$/$	17		76
				811	8	
10						
		5	$/$	7	6	

Once again, the original number was a perfect square and as there are three 'two-digit' groups, the square root has exactly three digits and so the calculation is complete, giving **576 as the square root of 331776**.

If, however, the original number was not a perfect square we would put a decimal point after the third digit and proceed in a similar fashion. So, just to verify that 576 is the square root of 331776 we will continue with the rest of the computation.

Step 6:

$$\text{G.D.} = 87 \text{ so N.D.} = 87 - \text{Duplex}(7\ 6) = 87 - (2 \times 7 \times 6) = 87 - 84 = 3$$

$$3 \div 10 = 0 \text{ rem } 3 \text{ i.e. quotient} = 0 \text{ and remainder} = 3$$

	33 / 1 7 7 6
10	811 83
	5 / 7 6 . 0

Step 7:

$$\text{The next and last G.D.} = 36 \text{ so N.D} = 36 - \text{Duplex}(7\ 6\ 0) = 36 - (2 \times 7 \times 0 + 6^2) = 36 - 36 = 0$$

$$0 \div 0 = 0 \text{ rem } 0 \text{ i.e. Q} = 0 \text{ and R} = 0$$

	33 / 1 7 7 6
10	811 83 0
	5 / 7 6 .0 0

The work is completed, 311776 is a perfect square!

If the given number is not a perfect square we will get the answer in decimal form.

Practical Method 2

Step1:

As before, we form distinct 'two-digit' groups from the right, and separate the last (far left) group with a forward slash, leaving a single blank row space for writing down the remainders, just as in the straight division method. After this, we draw a vertical line at the left side of the number. Here we will write the Divisor.

	33 / 17 76

Step 2:

In our example, the first group, located on the far left, separated by a forward slash, is 33. Again, we ask ourselves, what is the maximum square that can be subtracted from this group? The answer is 25. Now we write the square root of 25 or 5 below the horizontal line and this is the very first digit in the square root of the answer. The forward slash is written directly underneath. The Divisor which is exactly double the first digit of the square root, or in this case 2×5 or 10, is written to the left of the vertical line.

	33 / 17 76
10	
	5 /

Step 3:

As before, we write the remainder $33 - 25$ or 8 above the horizontal line, ahead of the 1, as in the straight division method.

	33 / 17 76
10	8
	5 /

Step 4:

The gross dividend (G.D.), after prefixing the remainder 8 to the 1, is now 81. Without subtracting anything from it, we simply divide the 81 by 10 giving: 8 remainder 1 which is also equivalent to 7 remainder 11. However, *at this moment in time, we will use the former quotient/remainder pair* i.e. $Q = 8, R = 1$

	33 / 17 76
10	8 1
	5 / 8

Step 5:

The next gross dividend (G.D.) is 17. Before dividing by 10 we find the net dividend (N.D.) by subtracting the duplex of the second square root digit (*in this example it is 8*) from the G.D.

i.e. $N.D. = 17 - \text{Duplex}(8)$

$$N.D. = 17 - 8^2 = 17 - 64 = -47 \text{ or } \overline{47}$$

Now, $-47 \div \square 10 = \overline{4} \text{ rem } \overline{7}$ i.e. $Q = \overline{4}$ and $R = \overline{7}$. We then write the quotient and remainder in their respective places.

	3 3 / 1 7 7 6
	8 1 $\overline{7}$
10	5 / 8 $\overline{4}$

Step 6:

G.D. = $\overline{77}$ so $N.D. = \overline{77} - \text{Duplex}(8 \overline{4}) = \overline{77} - (2 \times 8 \times \overline{4}) = \overline{77} - (\overline{64}) = -63 + 64 = 1$

So, $1 \div 10 = 0 \text{ rem } 1$ i.e. $Q = 0$ and $R = 1$

	3 3 / 1 7 7 6
	8 1 $\overline{7}$ 1
10	5 / 8 $\overline{4}$. 0

Step: 7

The next and last G.D. = 16 so $N.D. = 16 - \text{Duplex}(8 \overline{4} 0) = 16 - (2 \times 8 \times 0 + \overline{4}^2) = 16 - 16 = 0$

$0 \div 0 = 0 \text{ rem } 0$ i.e. $Q = 0$ and $R = 0$

	3 3 / 1 7 7 6
	8 1 $\overline{7}$ 1 0
10	5 / 8 $\overline{4}$. 0 0

Voila...there it is.

