Calculating Square Roots without a calculator ("Old Skool" Method)

The following algorithm for calculating square roots without a calculator was taught, back in the day, to keen maths students.

Algorithm for calculating Square Roots

Example 1: What is $\sqrt{758}$ to 3 decimal places?

1. First, we group the digits in pairs, from right to left, leaving either one or two digits on the left. For each pair of digits we will get one digit in the square root.

Square Root line	7	58

2. Now, find a number whose square is less than (or equal to) to left-most group of numbers. In this case, the left-most group of numbers only contains 7 and so the largest square less than or equal to 7 is 4. So, we write 2 above the square root line, as shown below.

	2	
Square Root line	7	58

3. We then square the 2, giving 4, and write that underneath the 7 and subtract it from the 7. Next we bring down the next pair of digits which, in this case, is 58.

	2	
Square Root line	7	58
	-4	
	3	58

4. Next, we double the number above the square root line and write it down in parenthesis with an empty space next to it.

	2	
Square Root line	7	58
	-4	
(47)	3	58

5. We then think of the single digit (mystery-digit1) that replaces the empty space, such that when mystery-digit1 is multiplied by forty-(mystery-digit1), it gives a result less than or equal to 358. After some experimenting we find that **mystery-digit_1 = 7,** because 47 x 7 = 329, which is less than but close to 358. We write the 7 above the square root line , next to the 2, and subtract the 329 from 358 leaving 29.

	2 7	
Square Root line	7	58
	-4	
(47)	3	58
	-3	29
		29

6. Next we bring down the next pair of digits. But in this case the digits are 00. Then we double the number above the square root line (27) and write this doubled number, (54) in parentheses with an empty space next to it as shown.

	2	7	
Square Root line	7	58	00
	-4		
(47)	3	58	
	-3	29	
(54_)		29	00

7. We then think of the single digit (mystery-digit2) that replaces the empty space, such that when mystery-digit2 is multiplied by five hundred and forty-(mystery-digit2), it gives a result less than or equal to 2900. After some experimenting we find that **mystery-digit_2 = 5,** because 545 x 5 = 2725, which is less than but close to 2900. We write the 5 above the square root line , next to the 27, and subtract the 2725 from 2900 leaving 175.

	2	7	.5	
Square Root line	7	58	00	00
	-4			
(47)	3	58		
	-3	29		
(545)		29	00	
		-27	25	
		1	75	

8. Next we bring down the next pair of digits, again in this case the digits are 00. Then we double the number above the square root line (275) and write this doubled number, (550) in parentheses with an empty space next to it as shown.

	2	7	.5	3	
Square Root line	7	58	00	00	
	-4				
(47)	3	58			
	-3	29			
(545)		29	00		
		-27	25		
(550_)		1	75	00	

9. We then think of the single digit (mystery-digit3) that replaces the empty space, such that when mystery-digit3 is multiplied by five thousand, five hundred and (mystery-digit3), it gives a result less than or equal to 17500. After some experimenting we find that **mystery-digit_3 = 3,** because 5503 x 3 = 16509, which is less than but close to 17500. We write the 3 above the square root line , next to the 27.5, and subtract the 16509 from 17500 leaving 991.

10. Next we bring down the next pair of digits, again in this case the digits are 00. Then we double the number above the square root line (2753) and write this doubled number, (5506) in parentheses with an empty space next to it as shown.

	2	7	.5	3	1	
Square Root line	7	58	00	00	00	
	-4					
(47)	3	58				
	-3	29				
(545)		29	00			
		-27	25			
(5503)		1	75	00		
		-1	65	09		
'(5506_)			9	91	00	
			-5	50	61	
			4	40	39	

11. We then think of the single digit (mystery-digit4) that replaces the empty space, such that when mystery-digit4 is multiplied by fifty-five thousand, and sixty and (mystery-digit3), it gives a result less than or equal to 99100. After some experimenting we find that **mystery-digit_4 = 1,** because 55061 x 1 = 55061, which is less than 99100. We write the 1 above the square root line , next to the 27.53, and subtract the 55061 from 99100 leaving 44039.

12. Next we bring down the next pair of digits, again in this case the digits are 00. Then we double the number above the square root line (27531) and write this doubled number, (55062) in parentheses with an empty space next to it as shown.

13. We then think of the single digit (mystery-digit5) that replaces the empty space, such that when mystery-digit5 is multiplied by five hundred and fifty thousand, six hundred and twenty and (mystery-digit5), it gives a result less than or equal to 4403900. After some experimenting we find that **mystery-digit_5 = 7,** because 550627 x 7 = 3854389, which is less than but close to 4403900. We write the 7 above the square root line , next to the 27.531, and subtract the 3854389 from 4403900 leaving 549511.

	2	7	.5	3	1	7
Square Root line	7	58	00	00	00	00
	-4					
(47)	3	58				
	-3	29				
(545)		29	00			
		-27	25			
(5503)		1	75	00		
		-1	65	09		
(55061)			9	91	00	
			-5	50	61	
(55062_)			4	40	39	00
			-3	85	43	89
				54	95	11

At this stage, we are going to cease doing any more calculations as the method has been clearly illustrated.

So, we can say that $\sqrt{758}$ is 27.532 to 3 decimal places.