Nikhilam Multiplication (Part 5)

So how about the case when we have to simultaneously multiply together four numbers that are all close to the base? Here is the logic:
Consider numbers N_1, N_2 , N_3 and N_4 all close to base x having respective deviations from the base of D_1, D_2, D_3 and D_4 which we will also call a, b, c and d.
So, we have the product of the four numbers, p $= N_1 \times N_2 \times N_3 \times N_4$

$$\begin{aligned} p &= (x + a)(x + b)(x + c)(x + d) \\ &= (x^2 + bx + ax + ab)(x^2 + cx + dx + ab) \\ &= x^4 + x^3(c + d) + x^2cd + x^3(a + b) + (a + b)(c + d)x^2 + cd(a + b)x \\ &\quad + abx^2 + ab(c + d)x + abcd \end{aligned}$$

Separately collecting together x^3, x^2, x and constant terms we have:

$$\begin{aligned} p &= x^3(x + a + b + c + d) + x^2(ab + ac + ad + bc + bd + cd) \\ &\quad + x(abc + abd + acd + bcd) + abcd \end{aligned}$$

So now we have four main sections to the answer: RHS, MIDDLE-RIGHT and MIDDLE-LEFT and LHS.

So, practically speaking, there will be four parts to the calculation:

- The RHS is the product of the Deviations: a, b, c and d, aka D_1, D_2, D_3 and D_4
- The MIDDLE-RIGHT section is the sum of all the individual products of any three Deviations taken from four
- The MIDDLE-LEFT section is the sum of all the individual products of any two Deviations taken from four
- The RHS is simply one number plus the Deviations of the other three numbers

Some examples are given below.

Example 1: (97 x 93 x 94 x 98)

In this first example all four numbers are near to the base 100 and the deviations are $\overline{3}$, $\overline{7}$, $\overline{6}$. and $\overline{2}$ the LHS is the product of the Deviations and is 252. Since we have a base possessing two zeros, there are only two digits allowed on the RHS and so we will carry over the 2 to the MIDDLE-RIGHT portion which is $\overline{3} \times \overline{7} \times \overline{6} + \overline{3} \times \overline{6} \times \overline{2} + \overline{3} \times \overline{7} \times \overline{2} + \overline{7} \times \overline{6} \times \overline{2} =$ **288**. The MIDDLE-LEFT is $\overline{3} \times \overline{7} + \overline{3} \times \overline{6} + \overline{3} \times \overline{2} + \overline{7} \times \overline{6} + \overline{7} \times \overline{2} + \overline{6} \times \overline{2}$ **= 113.** The LHS is simply one of the numbers plus the Deviations of the other three remaining numbers. So, let's us use $97 + \overline{7} + \overline{6} + \overline{2} =$ **82.**

eg.(1) 97 x 93 x 94 x 98					
Base = 100			97		$\overline{03}$
			93		$\overline{07}$
			94		$\overline{06}$
			98		$\overline{02}$
		82/	$_1$13/	$_2\overline{88}$	$_2$52
	'=	83/	11/	$\overline{86}$	52
	'=	83	10	14	52
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Nikhilam Multiplication of four numbers all near sub-base

So what about three numbers which are close to a sub-base? For instance how would we compute 307 x 306 x 309 x 312? First, lets go through the logic.

Let the sub-base be nx where n is some multiple, for instance 2, 3, 4 or even fractional for instance: ½, 1/5 etc. The analysis is similar to what we have previously done except that we are replacing x with nx. So, now $\mathbf{N_1 = nx + a, N_2 = nx + b, N_3 = nx + c}$, $\mathbf{N_3 = nx + d}$ and product p:

$$p = (nx + a)(nx + b)(nx + c)(nx + d)$$

$$= (n^2x^2 + x(na + nb) + ab)\,(n^2x^2 + x(nc + nd) + cd)$$

Grouping all the x^3, x^2, x and constant terms, we have:

$$p = n^4x^4 + an^3x^3 + bn^3x^3 + cn^3x^3 + dn^3x^3 + abn^2x^2 + acn^2x^2 + + adn^2x^2 + bcn^2x^2 + bdn^2x^2 + cdn^2x^2 + nxabc + nxabd + nxacd + nxbcd + abcd$$

$$= n^3x^3(nx + a + b + c + d) + n^2x^2(ab + ac + ad + bc + bd + cd) + nx(abc + abd + acd + bcd) + abcd$$

Fundamentally, we have a similar result as before, multiplying four numbers near the base, except that the middle-right term is multiplied by **n** and the middle-left term is multiplied by $\mathbf{n^2}$ and the LHS is multiplied by $\mathbf{n^3}$

Example 2: (307 x 306 x 309 x 312)

eg.(2) 307 x 306 x 309 x 312			307		07
Base = 100			306		06
Sub-Base = 300			309		09
RATIO = 3			312		12
		3^3 x (334)/	3^2 x (423)/	3 x (2286)/	$_{45}36$
	=	9018/	$_{38}07$/	$_{68}58$/	$_{45}36$
	=	9056	76	03	36
9056760336					