Nikhilam Multiplication (Part 4)

So how about the case when we have to simultaneously multiply together three numbers that are all close to the base. Here is the logic:
Consider numbers N_1, N_2 and N_3 all close to base x having respective deviations from the base of D_1, D_2 and D_3 which we will also call a, b and c.
So, we have the product of the three numbers, $p = N_1 \times N_2 \times N_3$

$$\begin{aligned} p &= (x + a)(x + b)(x + c) \\ &= (x + c)(x^2 + bx + ax + ab) \\ &= x(x^2 + ax + bx + ab) + c(x^2 + ax + ab + ab) \end{aligned}$$

Separately collecting together x^2, x and constant terms we have:

$$p = x^2(x + a + b + c) + x(ab + ac + bc) + abc$$

So, practically speaking, there will be three parts to the calculation:

- The RHS is the product of the Deviations: a, b and c, aka D_1, D_2, D_3
- The middle section is the sum of the individual products of pairs of Deviations
- The RHS is simply one number plus the Deviations of the other two numbers

Some examples are given below.

Example 1: (97 x 93 x 94)

In this first example all three numbers are near to the base 100 and the deviations are $\bar{3}$, $\bar{7}$ and $\bar{6}$. The product of the Deviations is $\bar{1}\bar{2}\bar{6}$ and since we have a base possessing two zeros, there are only two digits allowed on the RHS and so we will carry over the $\bar{1}$ to the MIDDLE portion. The Middle portion is $\bar{3} \times \bar{7} + \bar{3} \times \bar{6} + \bar{7} \times \bar{6} = \mathbf{81}$. The LHS is simply one of the numbers plus the Deviations of the other two remaining numbers. So, let's us use $97 + \bar{7} + \bar{6} = \mathbf{84}$.

eg.(1) 97 x 93 x 94				
Base = 100		97		$\bar{0}\bar{3}$
		93		$\bar{0}\bar{7}$
		94		$\bar{0}\bar{6}$
		84/	81/	$\bar{1}\bar{2}\bar{6}$
	=	84	80	$\bar{2}\bar{6}$
	=	84	79	74
847974				

Example 2: (108 x 109 x 92)

In this example we have two numbers slightly more than base 100 and the other number slightly below.

eg.(2) 108 x 109 x 92				
Base = 100		108		08
		109		09
		92		08
		109	64/	$_{5}$76
	=	109	69/	76
	=	108	30	24
1083024				

So what about three numbers which are close to a sub-base? For instance how would we compute 307 x 306 x 309? First, lets go through the logic.

Let the sub-base be nx where n is some multiple, for instance 2, 3, 4 or even fractional for instance: ½, 1/5 etc. The analysis is similar to what we have previously done except that we are replacing x with nx. So, now $\mathbf{N_1 = nx + a, N_2 = nx + b, N_3 = nx + c}$ and product p:

$$p = (nx + a)(nx + b)(nx + c)$$

$$= nx\,(n^2x^2 + x(na + nb) + ab\,) + c\,(n^2x^2 + x(na + nb) + ab\,)$$

Grouping all the x^2, x and constant terms, we have:

$$p = \mathbf{n^2}x^2(nx + a + b + c) + \mathbf{n}x(ab + ac + bc) + abc$$

Fundamentally, we have a similar result as before except that the middle term is multiplied by **n** and the LHS is multiplied by $\mathbf{n^2}$

Example 3: (212 x 208 x 196)

RHS = product of the Deviations: 12 x 8 x $\bar{4}$ = $\overline{384}$
MIDDLE term = 2 x (12 x 8 + 12 x $\bar{4}$ + 8 x $\bar{4}$) = 32
LHS = 2^2 x (212 + 8 + $\bar{4}$) = 4 x 216 = 864
Only two digits are allowed in the RHS and MIDDLE, so we carry over terms as usual.

eg.(3) 212 x 208 x 196				
Base = 100		212		12
Sub-Base = 200		208		08
Ratio = 2		196		04
		2^2 x 216/	2 x (16)/	$_{3}\overline{84}$
	=	864/	32/	$_{3}\overline{84}$
	=	864/	$3\bar{1}$	$\overline{84}$
8642816				