Nikhilam Multiplication (Part 3)

Up until now, we have concentrated on studying problems in which both of the numbers were near the same common base or sub-base. But if the numbers are both nearer to different bases or sub-bases then what do we do? Well, these problems are solved in a similar kind of way but with a slight modification. For now, we will show the method of solving these problems and then later on we will give a general mathematical explanation of how the method works.

Once again

- let $\mathbf{N_1}$ be the number that is near the base/sub-base $\mathbf{B_1}$
- let $\mathbf{N_2}$ be the number that is near the base/sub-base $\mathbf{B_2}$

$\mathbf{N_1 \times N_2}$ is computed as follows:

- As usual, we write down the numbers $\mathbf{N_1}$ and $\mathbf{N_2}$ and their deviations $\mathbf{D_1}$ and $\mathbf{D_2}$ from their respective bases/sub-bases.
- Write the chosen base/sub-base, in parentheses, to the right of each number i.e. $\mathbf{(B_1)}$ and $\mathbf{(B_2)}$.
- Next, we cancel an equal number of zeros from both $\mathbf{B_1}$ and $\mathbf{B_2}$; $\mathbf{B_1}$ is now temporarily transformed into $\mathbf{b_1}$ and $\mathbf{B_2}$ into $\mathbf{b_2}$.
- The RHS of the answer is, as usual, $\mathbf{D_1 \times D_2}$ and the number of digits in the RHS will be equal to the number of common zeros cancelled in each of the parentheses.

	N_1		D_1	(B_1)	$\rightarrow (b_1)$
	N_2		D_2	(B_2)	$\rightarrow (b_2)$
	$(N_1 b_2 + D_2 b_1)$	/	$D_1 \times D_2$		
Or					
	$(N_2 b_1 + D_1 b_2)$	/	$D_1 \times D_2$		

Example 1: (106 x 1009)

Bases $B_1 = 100$ and $B_2 = 1000$ are used and two common zeros are cancelled giving $b_1 = 1$ and $b_2 = 10$. The number of digits on RHS is two. The LHS = 106 x 10 + 9 x 1 OR 1009 x 1 + 6 x 10 giving 1069.

	106	6	/	$(B_1 = 1\cancel{00})$	$(b_1 = 1)$
	1009	9	/	$(B_2 = 10\cancel{00})$	$(b_2 = 10)$
Or	106 x 10 + 9 x 1 1009 x 1 + 6 x 10		/	54	
	=10069		/	54	
	=106954				

Example 2: (53 x 1012)

eg.(2)	(53 x 1012)				
	53	3	/	(B_1 = 50)	(b1 = 5)
	1012	12	/	(B_2 = 1000)	(b2 = 100)
Or	53 x 100 + 12 x 5 1012 x 5 + 3 x 100		/	$_{3}6$	
	=5360		/	$_{3}6$	
	=53636				

Example 3: (397 x 792)

Here we cancel two common zeros in the bases leaving $b_1 = 4$ and $b_2 = 8$, and so the RHS is of two digits.

eg.(3)	(397 x 792)				
	397	$\bar{3}$	/	(B_1 = 400)	(b_1 = 4)
	792	$\bar{8}$	/	(B_2 = 800)	(b_2 = 8)
Or	397 x 8 + $\bar{8}$ x 4 792 x 4 + $\bar{3}$ x 8		/	24	
	=3144		/	24	
	=314424				

Example 4: (34 x 1019)

eg.(4)	(34 x 1019)				
	34	4	/	(B_1 = 30)	(b_1 = 3)
	1019	19	/	(B_2 = 1000)	(b_2 = 100)
Or	34 x 100 + 19 x 3 1019 x 3 + 4 x 100		/	24	
	=3457		/	24	
	=345724				

Example 5: (257 x 4009)

We will solve this problem in two different ways

eg.(5) (257 x 4009)....Solution 1

	257	57	/	(B_1 = 200)	(b_1 = 2)
	4009	9	/	(B_2 = 4000)	(b_2 = 40)
Or	257 x 40 + 9 x 2 4009 x 2 + 57 x 40		/	$_5 13$	
	=10295		/	$_5 13$	
	=1030313				

eg.(5) (257 x 4009).....Solution 2

	257	7	/	(B_1 = 250)	(b_1 = 25)
	4009	9	/	(B_2 = 4000)	(b_2 = 400)
Or	257 x 400 + 9 x 25 4009 x 25 + 7 x 400		/	$_6 3$	
	=103025		/	$_6 3$	
	=1030313				

Example 6: (993 x 246)

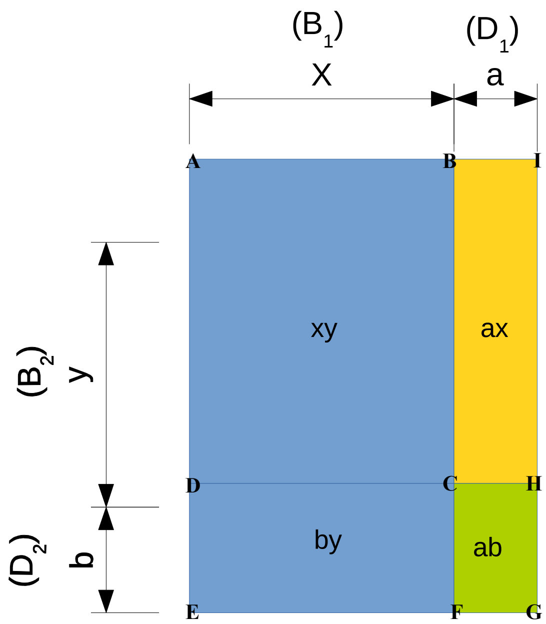
We will solve example 6 in two different ways

eg.(6) (993 x 246)...Solution 1

	993	$\bar{7}$	/	(B_1 = 1000)	(b_1 = 10)
	246	46	/	(B_2 = 200)	(b_2 = 2)
Or	993 x 2 + 46 x 10 246 x 10 + $\bar{7}$ x 2		/	$_3 \overline{22}$	
	=2446		/	$_3 \overline{22}$	
	=244278				

eg.(6)	(993 x 246)...Solution 2				
	993	$\bar{7}$	/	(B_1 = 1000)	(b_1 = 100)
	246	$\bar{4}$	/	(B_2 = 250)	(b_2 = 25)
Or	993 x 25 + $\bar{4}$ x 100 246 x 100 + $\bar{7}$ x 25		/	$_2 8$	
	=24425		/	$_2 8$	
	=244278				

Geometrical/Diagrammatical Explanation for Nikhilam Multiplication Part 3



Algebraic Explanation for Nikhilam Multiplication Part 3

$\mathbf{N_1 = x + a}$ and $\mathbf{N_2 = y + b}$; **x** is the base and **a** and **b** are deviations from the base, equivalent to D_1 and D_2 respectively. The product $\mathbf{N_1 \times N_2}$ is given by

(x + a) (y + b) = (xy + xb + ya + ab)

Rearranging gives:

(x + a) (y + b) is same as:

y(x + a) + xb + ab Or x(y + b) + ay + ab

Or put another way: $B_1N_2 + D_1B_2 + D_1D_2$ **Or** $B_2N_1 + D_2B_1 + D_1D_2$

Now, the practical working method that we have been using for multiplying two numbers near different bases aims to reduce the amount of computation by cancelling common zeros from both bases.
Here is the logic behind this:
Generally speaking we can say that:

$$B_1 = f \times 10^n$$

and

$$B_2 = g \times 10^m$$

where **f** and **g** are positive whole integers, as are **n** and **m**. In this analysis we will also assume that **m** > **n**.

So far, we have for the product, $p = N_1 \times N_2$

$$p = B_2N_1 + D_2B_1 + D_1D_2 \text{..................(NKf3)}$$

So next we define:

$$b_1 = B_1/10^n$$

and

$$b_2 = B_2/10^n$$

We are dividing both bases by the lowest common power of ten, which in our example is **10n**.

So now we have product:

$$p = b_2N_1 10^n + D_2b_1 10^n + D_1D_2$$

$$p = 10^n(b_2N_1 + b_1D_2) + D_1D_2 \text{..................(NKf3\_mod)}$$

As you can see, the 10^n outside of the parentheses, determines the number of digits which are placed in the right hand-side of the answer. Cancelling the common zeros actually reduces the amount of computation. However, bear in mind that we can still use the **NKf3** formula directly to compute the product.