Nikhilam Multiplication (Part 2)

Previously, we looked at multiplying together two numbers that were both near to the base but, what do we do when the numbers are not near to the base? In this case we make use of the sub-Sutra "Anurupyena" i.e proportionately. Essentially, this means that we have to choose a proportionate sub-base. And just what is a sub-base? A sub-base is a convenient multiple or sub-multiple of a base.
For example: 200, 300, 400 and 500 are sub-bases of 100 and are respectively 2, 3, 4 and 5 times the base 100.
Or, 40, 70 and 90 are the sub-bases of 10 which are respectively 4, 7 and 9 times the base 10.
Fifty is not only 5 times base 10 it is also ½ of base 100.

Now, the ratio of the sub-base to the base is simply known as the RATIO. Please note that sometimes we will refer to the sub-base as the working-base and will be using these terms interchangeably.

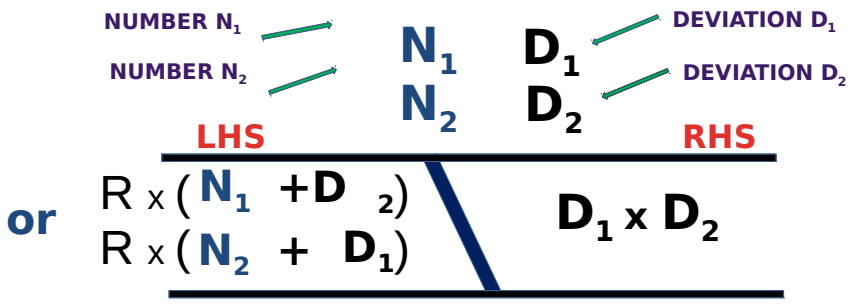
So we have **RATIO = sub-base : base**

Thus, for sub-base of 60 and a base of 10 the RATIO is 60: 10 or 6:1 or 6. For a sub-base of 50 and a base of 100 the RATIO is 50 : 100 = 1 : 2 or ½ .

The Method of multiplication is similar that outlined in Nikhilam Multiplication Part 1 but with some modifications:

1. Choose a suitable base and sub-base (a.k.a working-base)
2. The two numbers, $\mathbf{N_1}$ and $\mathbf{N_2}$, that are to be multiplied together, are written out one below the other.
3. Their deviations, $\mathbf{D_1}$ and $\mathbf{D_2}$ from the sub-base, are then written to the right of the numbers.
4. An answer box is formed which is divided in two using a forward slash: right hand side RHS and left hand side LHS. The RHS of the answer is formed by the product of the deviations i.e. $\mathbf{D_1}$ x $\mathbf{D_2}$.
5. The number of digits in the RHS of the answer will be equal to the number of zeros in the Base.
6. If the product of the deviations found in (4) above contains *fewer* digits than the number of zeros in the Base then an additional zero or zeros are added, placed to the left of the RHS, thus satisfying condition (5).
7. If the product of the deviations found in (4) above contains *more* digits than the number of zeros in the Base then the excess digit or digits are carried over to the LHS and added to LHS of the answer, after the complete part of the LHS has been found

The LHS of the answer is *simply* the **RATIO** multiplied by the sum of one number and the deviation of the other number i.e **R x (N_1 + D_2)** or **R x (N_2 + D_1)**.



Example 1: (53 x 56)
Let us solve this problem in two different ways

In the first solution we will use Base = 10 and Sub-Base = 50, giving a RATIO of 50 : 10 or 5

eg.(1) 53 x 56....solution 1

Base = 10

Sub-Base = 50

RATIO = 5

	53	+	3	
	56	+	6	
	5 x (59)	/	$_1$8	
=	295	/	$_1$8	
=	2968			

2968

In the second solution we will use Base = 100 and Sub-Base = 50, giving a RATIO of 50 : 100 or ½.
Please note: On the LHS, the ½ is equivalent to 50 units and those units are added back onto the RHS.

eg.(1) 53 x 56....solution 2

Base = 100

Sub-Base = 50

RATIO = ½

	53	+	03	
	56	+	06	
	½ x (59)	/	18	
=	29 $_{1/2}$	/	18	
=	2968			

2968

Example 2: (42 x 47)
Let us solve this problem in three different ways

In the first solution we will use Base = 10 and Sub-Base = 40, giving a RATIO of 40 : 10 or 4

eg.(2) 42 x 48....solution 1

Base = 10

Sub-Base = 40

RATIO = 4

	42	+	2	
	47	+	7	
	4 x (49)	/	$_1$4	
=	196	/	$_1$4	
=	1974			

1974

In the second solution we will use Base = 10 and Sub-Base = 50, giving a RATIO of 50 : 10 or 5

eg.(2) 42 x 48....solution 2

Base = 10

Sub-Base = 50

RATIO = 5

		42	+	$\overline{8}$
		47	+	$\overline{3}$
		5 x (39)	/	$_2 4$
	=	195	/	$_2 4$
	=	1974		

1974

In the third solution we will use Base = 100 and Sub-Base = 50, giving a RATIO of 50 : 100 or ½
Please note: Once again we have to convert the ½ unit on the LHS to 50 units on the RHS.

eg.(2) 42 x 48....solution 3

Base = 100

Sub-Base = 50

RATIO = ½

		42	+	$\overline{08}$
		47	+	$\overline{03}$
		½ x (39)	/	24
	=	$19_{1/2}$	/	24
	=	1974		

1974

Example 3: (4879 x 4993)

Let us solve this problem in two different ways

eg.(3) 4879 x 4993....solution 1

Base = 1000

Sub-Base = 5000

RATIO = 5

		4879	+	$\overline{121}$
		4993	+	$\overline{007}$
		5 x (4872)	/	847
	=	24360	/	847
	=	24630847		

24630847

eg.(3) 4879 x 4993....solution 2

Base = 10000

Sub-Base = 5000

RATIO = ½

		4879	+	$\overline{0121}$
		4993	+	$\overline{0007}$
		½ x (4872)	/	0847
	=	2436	/	0847
	=	24630847		

24630847

Here are some more examples in which both the numbers, that is, the multiplicand and the multiplier, are both near to the same sub-base.

Example 4: (249 x 257)

eg.(4) 249 x 257

Base = 1000

Sub-Base = 250

RATIO = ¼

	249	+	$\overline{001}$
	257	+	007
	¼ x (256)	/	'$\overline{007}$
=	64	/	'$\overline{007}$
=	63997		

63397

Example 5: (4859 x 5007)

eg.(5) 4859 x 5007

Base = 10000

Sub-Base = 5000

RATIO = ½

	4859	+	$\overline{0141}$
	5007	+	0007
	½ x (4866)	/	'$\overline{0987}$
=	2433	/	'$\overline{0987}$
=	24329013		

24329013

Example 6: (1985 x 2017)

eg.(6) 1985 x 2017.....solution 1

Base = 1000

Sub-Base = 2000

RATIO = 2

	1985	+	$\overline{015}$
	2017	+	017
	2 x (2002)	/	'$\overline{255}$
=	4004	/	'$\overline{255}$
=	4003745		

4003745

And eg.(6) now Solution 2

eg.(6) 1985 x 2017.....solution 2

Base = 10000

Sub-Base = 2000

RATIO = 1/5

	1985	+	$\overline{0015}$
	2017	+	0017
	1/5 x (2002)	/	'$\overline{0255}$
=	400 2/5	/	'$\overline{0255}$
=	4003745		

4003745

Algebraic Explanation for Nikhilam Multiplication Part 2

$N_1 = nx + a$ and $N_2 = nx + b$; **x** is the base and **a** and **b** are deviations from the base and **n** is a multiple of the base. We can write the product of the numbers, $N_1 \times N_2$:

$(nx + a)(nx + b) = (n^2x^2 + nax + nbx + ab)$

After some re-arranging we get: $nx(nx + a + b) + ab$

Please note:

- The term **ab** is the product of the deviations
- The term **nx + a + b** is one number plus the deviation of the other number
  ie. **(nx + a) + b** or **(nx + b) + a**
- Once again, the **x** outside of the bracket has the effect of moving the quantity **(nx + a + b)** to the left as many places as there are zeros in the base; the quantity is then multiplied by **n** which is the same as the **RATIO** defined previously.