Nikhilam Multiplication (Part 1)

Nikhilam Multiplication is specifically used when the numbers that are to be multiplied together are close to a particular base such as 10, 100, 1000 etc.

In Nikhilam Multiplication we use the Vedic Maths sutra number 2: Nikhilam Navatascharamam Dashatah (***All from 9 last from 10***). The special thing about using this method is that we don't actually divide by the dividend at all! The answer is obtained by repeated digit to digit multiplication followed by addition. The Quotient(Q) and the Remainder(R) are obtained side by side.

Nikhilam Multiplication Method

1. The two numbers, $\mathbf{N_1}$ and $\mathbf{N_2}$, that are to be multiplied together, are written out one below the other.
2. Their deviations, $\mathbf{D_1}$ and $\mathbf{D_2}$, are then written to the right of the numbers. *If a number is below the Base the deviation will be negative and will be known as a deficiency. If a number is above the Base the deviation will be positive and will be known as a surplus.*
3. An answer box is formed which is divided in two using a forward slash: right hand side RHS and left hand side LHS. The RHS of the answer is formed by the product of the deviations i.e. $\mathbf{D_1 \times D_2}$.
4. The number of digits in the RHS of the answer will be equal to the number of zeros in the Base.
5. If the product of the deviations found in (4) above contains *fewer* digits than the number of zeros in the Base then an additional zero or zeros are added, placed to the left of the RHS, thus satisfying condition (5).
6. If the product of the deviations found in (4) above contains *more* digits than the number of zeros in the Base then the excess digit or digits are carried over to the LHS and added to LHS of the answer, after the complete part of the LHS has been found.
7. The LHS of the answer is *simply* the sum of one number and the deviation of the other number i.e $\mathbf{N_1 + D_2}$ or $\mathbf{N_2 + D_1}$.

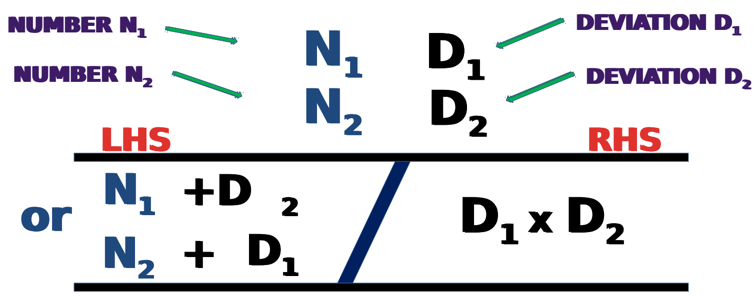

Fig.1

The number $\mathbf{N_1}$ is called the multiplicand and $\mathbf{N_2}$ is called the multiplier. The product is the multiplicand x multiplier.

Example 1 (103 x 108)
Step 1:
In our first example, both 103 and 108 are near to base 100, so we can use this as our base.
The RHS will have two digits, as there are two zeros in the base.
The Deviations, $\mathbf{D_1}$ and $\mathbf{D_2}$ are 03 and 08 respectively, giving: $\mathbf{D_1 \times D_2} = 03 \times 08 = 24$ (**RHS**)
Step 2:
$\mathbf{N_1} + \mathbf{D_2} = 103 + 08 = 111$
OR
$\mathbf{N_2} + \mathbf{D_1} = 108 + 03 = 111$ (**LHS**)
Step 3:
Take away forward slash, giving Product = 1 1 1 2 4

eg.(1) 103 x 108			
Base = 100			
	103	+	03
	108	+	08
	111	/	24
11124	LHS		RHS

Here are three more examples in which **both numbers are ABOVE the base**

Example 2 (13 x 12)

eg.(2) 13 x 12			
Base = 10			
	13	+	3
	12	+	2
	15	/	6
156			

Example 3 (114 x 107)

eg.(3) 114 x 107			
Base = 100			
	114	+	14
	107	+	07
	121	/	98
12198			

Example 4 (1357 x 357)

In this example the RHS initially has four digits, where only three are allowed, and so the excess digit is carried over to the LHS.

eg.(4) 1357 x 1008				
Base = 1000				
		1357	+	357
		1008	+	008
		1365	/	$_{2}856$
	=	1367		856
1367856				

Here are three examples in which **both numbers are BELOW the base**

Example 5 (97 x 95)

eg.(5) 97 x 95				
Base = 100				
		97	+	$0\overline{3}$
		95	+	$0\overline{5}$
		92	/	15
9215				

Example 6 (978 x 993)

eg.(6) 978 x 993				
Base = 1000				
		978	+	$\overline{22}$
		993	+	$0\overline{7}$
		971	/	154
971154				

Example 7 (89 x 83)

eg.(7) 89 x 83				
Base = 100				
		89	+	$\overline{11}$
		83	+	$\overline{17}$
		72	/	$_{1}87$
	=	73	/	87
7387				

Here are three examples in which one **numbers is BELOW the base and the other number is ABOVE the base.**
In these examples, the RHS is initially negative as the product of a negative deviation with a positive deviation always gives a negative number producing a Vinculum number. To produce the final answer, the Vinculum number is normalised i.e. transformed into a normal number.

Example 8: (87 x 107)

eg.(8) 87 x 107					
Base = 100					
		87	+	$\overline{13}$	
		107	+	07	
		94	/	$\overline{91}$	
	=	93	/	09	
9309					

Example 9: (1011 x 992)

Here the RHS initially has only two digits and so we add an extra zero to make the total number of digits equal to the number of zeroes in the base. Then we normalise.

eg.(9) 1011 x 992					
Base = 1000					
		1011	+	11	
		992	+	$0\overline{8}$	
		1003	/	$0\overline{88}$	
	=	1002	/	912	
1002912					

Example 10: (1011 x 992)

eg.(10) 116 x 85					
Base = 100					
		116	+	16	
		85	+	$\overline{15}$	
		101	/	$_2\overline{40}$	
	=	99	/	$\overline{40}$	
	=	98	/	60	
9860					

As usual, if required, we can check the all results using the Beejank Vedic Check Method. We will check eg.(1). 103 x 108 = 11124

Digit sum(103) = 1 + 0 + 3 = 4 So Bj(103) = 4

Digit sum(108) = 9 So Bj(108) = 0

Product of Beejanks is 4 x 0 = 0.

Digit sum(11124) = 1 + 1 + 1 + 2 + 4 = 9 So Bj(11124) = 0

Hence the product of Beejanks of the multiplicand (103) and multiplier (108) is the same as the Beejank of product (11124). We can conclude that the calculation is probably correct.

Algebraic Explanation for Nikhilam Multiplication

The two numbers, $\mathbf{N_1}$ and $\mathbf{N_2}$ are both near to base **x** where $\mathbf{N_1 = (x + a)}$ and $\mathbf{N_2 = (x + b)}$ and **a** and **b** are respective deviations of the numbers from the base. We can write the product of the numbers as: $\mathbf{N_1 \times N_2}$
Or
$\mathbf{(x + a)(x + b) = x^2 + ax + bx + ab}$

After some re-arranging we get: $\mathbf{x (x + a + b) + ab}$

Please note:

- The term **ab** is the product of the deviations
- The term **x + a + b** is one number plus the deviation of the other number ie. **(x + a) + b** or **(x + b) + a**
- The x outside of the bracket has the effect of moving the quantity **(x + a + b)** to the left as many places as there are zeros in the base.

Geometrical/Diagrammatical Explanation for Nikhilam Multiplication

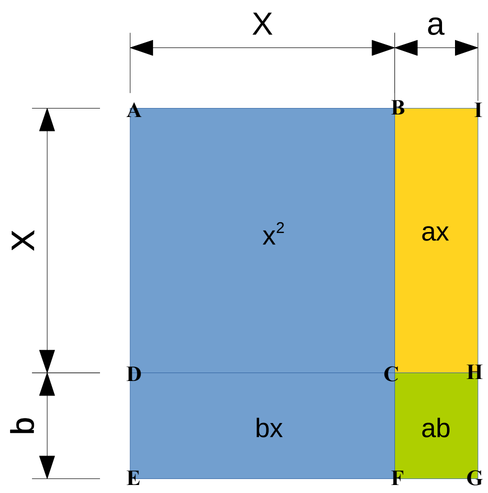

Fig.2

In the diagram, we draw a square of side length **x** and this represents the base i.e. base = 10, 100, 1000 etc.

As before, N_1 is **(x + a)** and N_2 is **(x + b)** and as we are interested in finding out the product of these numbers we just need to find the whole area AEGI, which is once again: $\mathbf{x^2 + ax + bx + ab}$

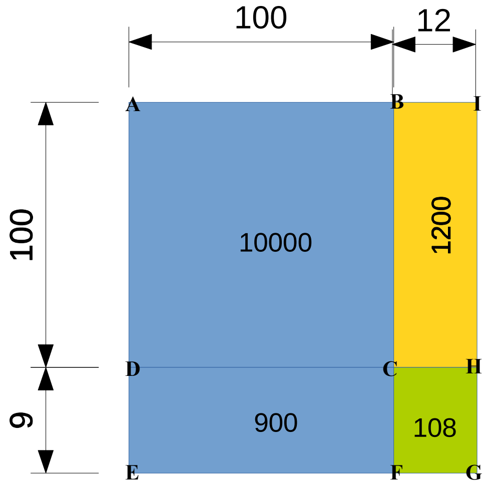

Fig.3

In this example we want to find the product 112 x 109. Please note that the deviations are both positive ie. Surpluses. The work out this problem we simply calculate the area of AEGI.

Area AEGI = 10000 + 1200 + 900 + 108

= 12208

Now solve some questions using the Nikhilam Multiplication Method. (See Question Sheet)